SPARSE REPRESENTATIONS WITH CONE ATOMS

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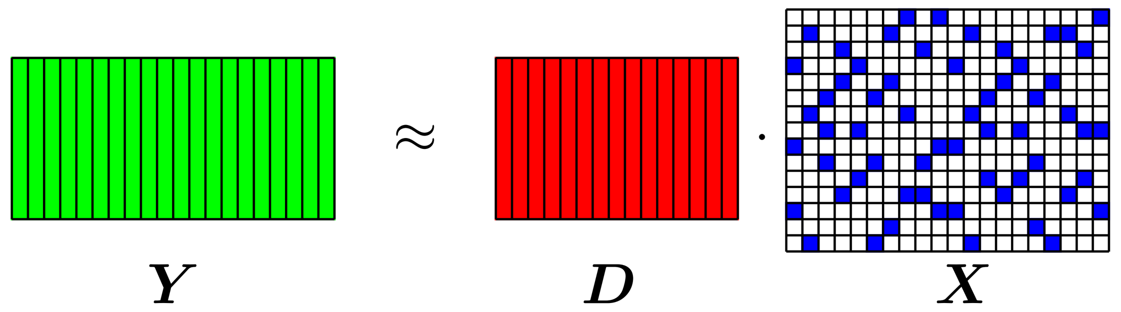
SPARSE REPRESENTATION

- Data set (signals) $\boldsymbol{Y} \in \mathbb{R}^{m \times N}$
- dictionary $\boldsymbol{D} \in \mathbb{R}^{m \times n}$
- sparsity level s
- Find sparse representations $\boldsymbol{X} \in \mathbb{R}^{n \times N}$



CONE ATOMS

- **Idea**: extend the atom from a single vector to an infinite set
- Particular case: cone atoms
- Central vector $\boldsymbol{d} \in \mathbb{R}^m$, with $\|\boldsymbol{d}\| = 1$
- Radius ρ

$$\mathcal{C}(\boldsymbol{d}, \rho) = \{\boldsymbol{a} \in \mathbb{R}^m \mid \|\boldsymbol{a}\| = 1, \|\boldsymbol{a} - \boldsymbol{d}\| \leq \rho\}$$

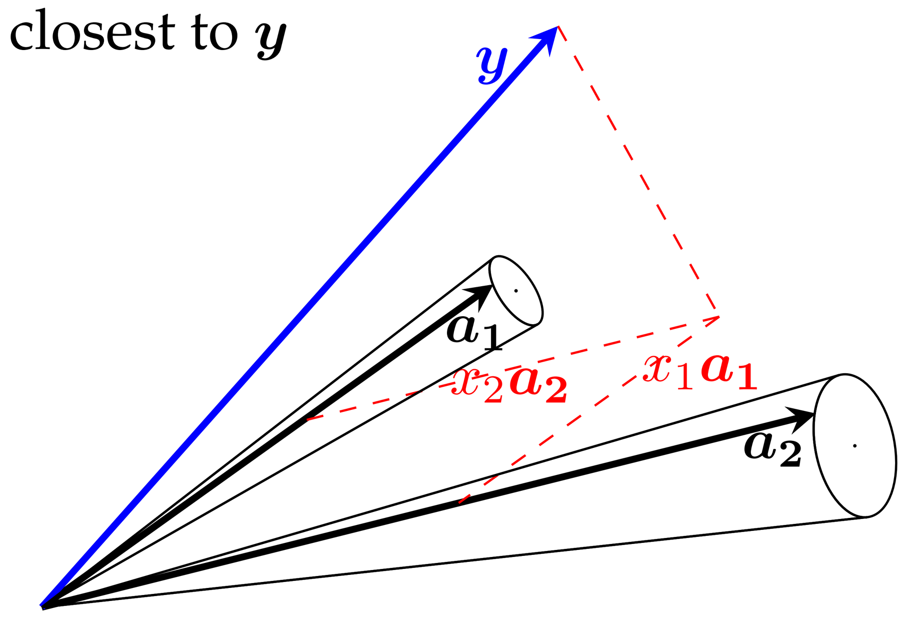
SPARSE REPRESENTATION WITH CONE ATOMS

Optimization problem

$$\begin{array}{ll} \min\limits_{\boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{a}_j \in \mathbb{R}^m} & \|\boldsymbol{y} - \sum_{j=1}^n \boldsymbol{a}_j x_j\|_2 \\ \text{s.t.} & \|\boldsymbol{x}\|_0 \leq s \\ & \boldsymbol{a}_j \in \mathcal{C}(\boldsymbol{d}_j, \rho_j), \ j = 1:n \end{array}$$

- $\boldsymbol{a}_j$ – actual atoms

- The plane generated by $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ is the closest to $\boldsymbol{y}$

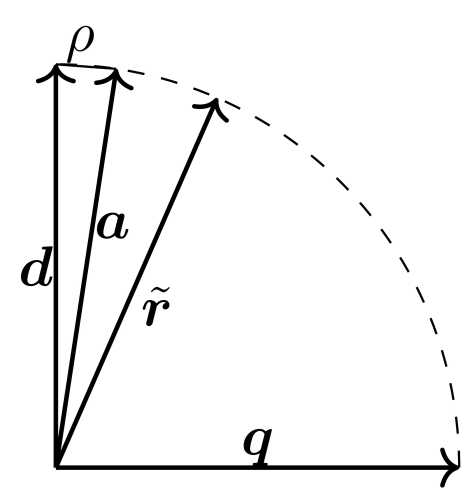


NEAREST CONE ATOM

Given $\tilde{\boldsymbol{r}} \in \mathbb{R}^m$, with $\|\tilde{\boldsymbol{r}}\| = 1$, find nearest atom from given cone $\mathcal{C}(\boldsymbol{d}, \rho)$:

$$\begin{array}{ll} \min\limits_{\boldsymbol{a} \in \mathcal{C}(\boldsymbol{d}, \rho)} & \|\boldsymbol{a} - \tilde{\boldsymbol{r}}\|_2 \\ \text{s.t.} & \|\boldsymbol{a}\| = 1 \end{array}$$

Best atom: projection on cone. 2D problem



CONE-OMP

- The usual greedy structure
- Current residual is $\boldsymbol{r}$
- Next atom
  - nearest projection of $\boldsymbol{r}$ on a cone
  - simple formula using $\boldsymbol{D}^T \boldsymbol{r}$
- Least squares on current support
  - actual atoms are not fixed!
  - few rounds of coordinate descent
  - all atoms but $\boldsymbol{a}_j$ are fixed
  - new $\boldsymbol{a}_j$ is projection of residual on $\mathcal{C}(\boldsymbol{d}_j, \rho_j)$
- Overall complexity
  - a few times that of OMP

FALSE POSITIVE RESULTS

- Averages over 10 sets of dictionaries
- Adler et al: FP=13 at true positive rate 97.18% (a bit less than TP=39)

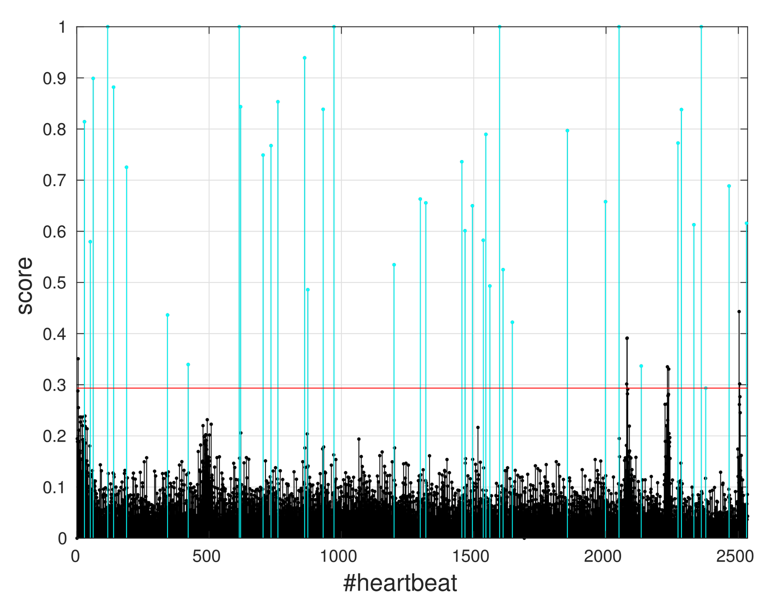
	n	s	ρ	ROC AUC	FP TP=40	FP TP=39	FP TP=38
OMP	96	3	-	0.99971	10.3	6.4	4.2
	96	4	-	0.99971	11.7	6	3.6
	128	4	-	0.99968	10.2	6.7	4.9
Cone-OMP	64	3	0.08	0.99984	6.7	4.7	2.4
	96	3	0.05	0.99980	6.3	4	2.8
	64	4	0.07	0.99986	5.7	2.8	1.8
	96	4	0.05	0.99989	6.7	3	0.9
	128	4	0.05	0.99977	8.3	3.7	2.8

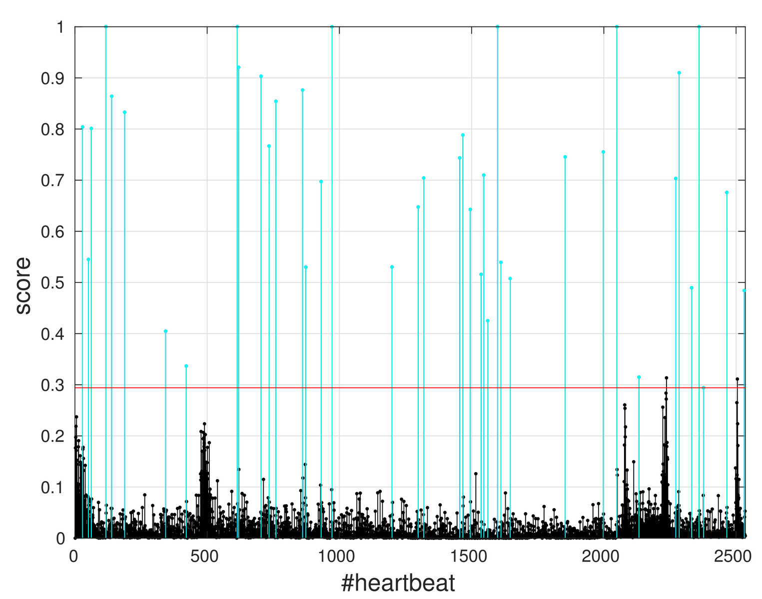
ARRHYTHMIA DETECTION

- MIT-BIH arrhythmia database, rec. #109
  - 360 Hz, about 30 minutes
  - 2530 heartbeats, 40 anomalies
- Extract all windows of length 256 and reduce the dimension to 32 via PCA
- 6 segments, $\sim$ 108000 signals each
- Train a dictionary for each segment
- Get OMP and Cone-OMP errors
- Anomaly score: median error for each heartbeat, from 100 windows left and 100 right of the R point

REPRESENTATIVE ERRORS

OMP





Cone-OMP

- black=normal, cyan=anomaly, red=smallest error of an anomaly
- Cone-OMP represents very well the normal beats

CONCLUSIONS

- Cone atoms: infinite set instead of vector
- Cone-OMP: greedy sparse representation for dictionaries with cone atoms
- Only a few times slower than OMP
- Good results in anomaly detection
- Future work
  - Dictionary learning with cone atoms
- Bibliography: A. Adler, M. Elad, Y. Hel-Or, and E. Rivlin, Sparse coding with anomaly detection, J. Signal Proc. Syst., vol.79, no.2, pp.179-188, 2015